Application of Eigenvector Centrality in Ranking Fan-Favorite Characters from Love Live! Nijigasaki High School Idol Club

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***Abstract*—Love Live! Nijigasaki High School Idol Club is a Japanese multimedia project centered on a virtual idol group, prominently featured in its television anime adaptation with the same name and the mobile game *Love Live! School Idol Festival: ALL STARS.* As part of a well-known franchise with an active fanbase, from 2017 to 2019, the project conducted monthly popularity polls for its nine main characters. This paper presents an analysis of the popularity poll results using the eigenvector centrality method. The poll results were transformed into a weighted, directed graph in an adjacency matrix representation of how favorable characters score against one another, presented in two variations for the experiment. Compared to a ranking result from the Borda Count method, one variation showed a balanced score distribution while the other showed a highly skewed score distribution. Through this analysis, it is determined that Setsuna Yuki, Karin Asaka, and Kasumi Nakasu are the top three fan-favorite characters, respectively.**

***Keywords*—Eigenvector, Eigenvector centrality, Love Live! Nijigasaki High School Idol Club, Popularity polls**

I. Introduction

Love Live! is a Japanese multimedia project developed by ASCII Media Works' *Dengeki G's* magazine, *Lantis* music label, and *Sunrise* anime studio. Since the beginning of the project on June 30, 2010, Love Live! has released several forms of content, which include music CDs, anime music videos, a manga adaptation, video games, card games, and an anime series, with millions of dollars of revenue across those media forms. The franchise's popularity helped define the genre of anime idols and its popularization in Japan and worldwide. The franchise developed from a single idol group called μ's (pronounced "Muse") that debuted in 2010, followed by several other groups, namely Aqours which debuted in 2015, Nijigasaki High School Idol Club in 2017, Liella! in 2020, and Hasunosora Girls' High School Idol Club in 2023 [1].

Some of the main compelling factors of the franchise are its virtual idol characters and its interaction with its fanbase. The characters each have unique designs and charming personalities and dynamics between them. It is enhanced more by *Sunrise*'s high-quality animation and the voice actors' engaging dub [2]. Furthermore, the franchise maintains a good relationship with its fanbase by holding live performances, live broadcasts, selling merchandise, etc [1]. One of the fan interactions unique to Love Live! Nijigasaki High School Idol Club is its monthly popularity polls that were conducted from 2017 to 2019. Each month, fans can vote on which characters are their favorite and view the monthly ranking on the news section of the project's official website [3].

The monthly popularity poll conducted for Nijigasaki High School Idol Club provides an opportunity to analyze fan preferences through quantitative methods. By treating it as a network of character relationships, we can apply eigenvector centrality analysis, a commonly used ranking method, to see how characters compare against one another. The resulting ranking of characters might then be used for the project's further consideration, i.e. enhancing the characters' qualities, merchandise distribution, and live performance/broadcast lineup.

Furthermore, this study will explore the eigenvector centrality of different graph variations. Specificallly, this study attemps to analyze how different graph variations influence both eigenvector centrality values and the ranking result. The ranking results will provide an insight into which graph representation for the eigenvector centrality method is best suited for a given dataset and scenario.

II. Theoretical Framework

A. Directed Weighted Graphs

Graphs are discrete structures that consist of vertices that are connected by edges. A directed weighted graph is a type of graph in which edges have a specific direction and are given a numerical value called weight. A directed weighted graph with weights w can be represented in an adjacency matrix A with properties as follows.

$$A = [a_{i,j}]$$

$$a_{i,j} = \begin{cases} w_{i,j} \text{ } if\ there\ exists\ a\ vertex\ from\ edge\ i\ to\ j \\ 0\ otherwise \end{cases}$$

[4]

Directed weighted graphs can be used for several purposes, including representing connections between character rankings. Suppose a directed weighted graph and its corresponding adjacency matrix shown in Figure 1.

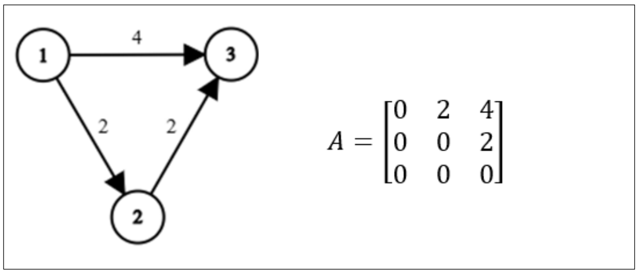

Figure 1. Directed Weighted Graph and Its Adjacency Matrix
Source: Author

In the graph shown above, each vertex represents a single character and the edge between characters represents how favorably a character scores. For example, character 1 scores two points more than character 2 and four points more than character 3. Furthermore, character 2 scores two points more than character 3.

B. Eigenvalues and Eigenvectors

Given an $n \times n$ matrix A, a non-zero vector $\boldsymbol{x}$, and a scalar λ, $\boldsymbol{x}$ and λ are respectively called the eigenvector and eigenvalue of A if and only if $A\boldsymbol{x} = \lambda\boldsymbol{x}$. In a geometrical sense, the eigenvector $\boldsymbol{x}$ represents a column matrix in which the multiplication of an $n \times n$ matrix with $\boldsymbol{x}$ is a factor of the eigenvector itself.

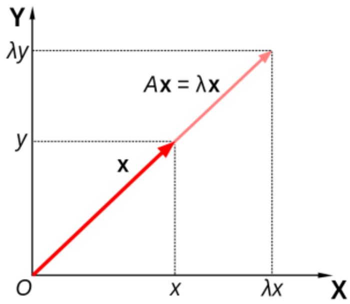

Figure 2. Eigenvector Illustration
source: https://informatika.stei.itb.ac.id/~rinaldi.munir/

To determine the eigenvalues and eigenvectors of a matrix, the equation can then be modified as follows.

$$A\boldsymbol{x} = \lambda\boldsymbol{x}$$
$$IA\boldsymbol{x} = \lambda I\boldsymbol{x}$$
$$A\boldsymbol{x} = \lambda I\boldsymbol{x}$$
$$(\lambda I - A)\boldsymbol{x} = 0 \;\dots\; (1)$$

For the equation to have a non-trivial solution ($\boldsymbol{x} \neq \boldsymbol{0}$), it must satisfy the characteristic equation $det(\lambda I - A) = 0$. The solution for said equation, which are $\lambda_1, \lambda_2, \dots, \lambda_n$, are the eigenvalues of the given matrix. By substituting each eigenvalue to (1), we can then determine the eigenvector for each corresponding eigenvalue [5].

C. Centrality and Eigenvector Centrality

In graph theory and network analysis, centrality is a metric that measures the degree to which an entity is central to a graph or a network. With centrality, we can identify key or important entities, including analyzing the importance of each entity in a given graph. Eigenvector centrality, also known as eigencentrality, is one of the main measures used in graph theory and network analysis. Eigenvector centrality measures how important an entity is, which is represented as a vertex of a graph, based on the weight of the edge that connects it with other entities. This measurement acknowledges that some connections might be more important than others. In general, connections to entities with higher importance will have more influence rather than connections with lower importance.

Suppose vertex i has an eigenvector centrality x_i. Given an adjacency matrix $A_{i,j}$, the eigenvector centrality of i can be denoted as

$$x_i = \frac{1}{\lambda}\sum_{j=1}^{n} A_{ij}x_j$$

in which λ is a constant. Define a vector $\boldsymbol{x} = (x_1, x_2, \dots, x_n)$, the equation can be rewritten as follows:

$$\lambda\boldsymbol{x} = A\boldsymbol{x}$$

Thus, the eigenvector centrality $x_1, x_2, \dots, x_n$, represented in vector $\boldsymbol{x}$, is simply an eigenvector of an adjacency matrix A with eigenvalue λ. For the eigenvector centrality to be non-negative, according to the Perron-Forbenius theorem, it is said that λ must be the largest eigenvalue of the corresponding adjacency matrix with $\boldsymbol{x}$ as its eigenvector [6].

Eigenvector centrality is a useful measure that might reveal some insights which other centrality methods overlook. For example, a variation of eigenvector centrality, called PageRank, is used by Google to rank web pages and provides satisfying results for said problem [6].

D. Borda Count

Borda Count is a voting or ranking method named after Jean-Charles de Borda who developed the method in 1770. Sometimes referred to as a consensus-based voting system, Borda Count is done by assigning points to candidates based on their ranking. For example, a candidate in the last place will get one point, a candidate in the second-to-last place will get two points, and so forth [7]. Mathematically, in a Borda Count with n candidates, if a ranking of a candidate is r, the point or score for the corresponding candidate, denoted by s, is formulated as follows:

$$s = n - (r - 1) \dots \;(2)$$

The points are all then accumulated to a single result. Winners are ordered from candidates with the largest accumulated points.

E. Love Live! Nijigasaki High School Idol Club

Love Live! Nijigasaki High School Idol Club is a Japanese multimedia project, part of the Love Live! franchise, which centered on a virtual idol group called Nijigasaki High School Idol Club. The group consists of nine initial members, namely Ayumu Uehara, Kasumi Nakasu, Setsuna Yuki, Ai Miyashita, Emma Verde, Rina Tennoji, Kanata Konoe, Shizuku Osaka, and Karin Asaka. Since its first started in 2017, the project has produced six albums, several singles and collabs, seven live performances, television and movie anime, and many other contents from all sorts of forms of media [8].

Figure 3. Love Live! Nijigasaki High School Idol Club Characters
source: https://en.wikipedia.org/wiki/Love_Live!_Nijigasaki_High_School_Idol_Club_(TV_series)

One notable content from Love Live! Nijigasaki High School Idol Club is the monthly popularity polls that were conducted regularly from 2017 to 2019. In said popularity polls, fans were able to vote for their favorite character and view the result, which is the corresponding month's characters' ranking, on the news section of the project's official website. In total, thirty popularity polls that have been conducted for the nine initial members [3].

Figure 4. Announcement of the 16^{th} *Popularity Poll Results*
source: https://lovelive-as.bushimo.jp/vote/v201810/

III. Data Collection and Preparation

A. Data Collection

The dataset used in this paper consists of ten columns and thirty rows, with the first column being a number to denote the order of the popularity polls, nine columns for each member, and each row representing a character's ranking for a popularity poll. The popularity poll data was collected from the Love Live! fansite which summarizes the result of each popularity poll from Love Live! Nijigasaki High School Idol Club official website [9]. The data was stored in an Excel spreadsheet (**.xlsx*) format as shown in Table 1 below. The full version of the dataset can be found in Appendix A.

Table 1. Dataset Excel Spreadsheet Format

No.	Ayumu Uehara	⋯	Karin Asaka
1.	4	⋯	5
⋮	⋮	⋱	⋮
30.	1	⋯	8

B. Data Preparation

In this experiment, there will be three variations in the ranking method, which are Borda Count and eigenvector centrality on two graph variations. Borda Count is used as a base comparison for the eigenvector centrality results; to see how different the results are compared to a simple ranking method. With this comparison, we can check whether eigenvector centrality can capture new insights or information given in the same dataset. On datasets with ordinal values such as rankings, Borda Count can be done by first converting the rankings to a score. Given that the dataset consists of nine characters, the scoring of each ranking according to (2) can be denoted as

$$s = 9 - (r - 1)$$
$$s = 10 - r \dots (3)$$

where r is a ranking of a character in a popularity poll and s as its corresponding score. For a character with score $s_1, s_2, \dots, s_{30}$, the Borda Count for the corresponding character, s_{total}, is calculated with the formula shown below.

$$s_{total} = \sum_{i=1}^{30} s_i \quad \dots \quad (4)$$

Define Mr as the matrix of popularity poll rankings, Ms as the matrix of popularity poll scores, and arr_borda as the array that holds the Borda Count of characters (initialized as a zero array), the process given by (3) and (4) can be done with the following pseudocode written in algorithmic notation.

```
i traversal[0..30]
  j traversal[0..9]
      Ms[i][j] ← 10 - Mr[i][j]
      arr_borda[j] ← arr_borda[j] + Ms[i][j]
```

The first graph variation for the eigenvector centrality method is a directed weighted graph that represents how many times a character has a greater score compared to each other. For example, suppose a three-character, three-popularity-poll score matrix as shown in Table 2.

Table 2. Popularity Poll Score Matrix Example

No.	A	*B*	C
1.	3	2	1
2.	2	3	1
3.	3	1	2

The adjacency matrix for the given score matrix is as follows:

$$A = \begin{bmatrix} 0 & 2 & 3 \\ 1 & 0 & 2 \\ 0 & 1 & 0 \end{bmatrix}$$

For example, $A_{2,1}$ represents how many times character B has a greater score compared to character A, $A_{2,3}$ represents how many times character B has a greater score compared to character C, and so forth. In an algorithmic notation pseudocode, the process is written as shown below (assuming that adjacency matrix A is initialized as a zero matrix).

```
i traversal[0..30]
    j traversal[0..9]
        k traversal[0..9]
            if Ms[i][j] > Ms[i][k] then
                A[j][k] ← A[j][k] +1
```

The second graph variation for the eigenvector centrality method is a directed weighted graph that represents the difference between the two characters' Borda Count results. If a character has a greater Borda Count than another character, the former's vertex will have a directed edge to the latter's vertex with the Borda Count's difference as its weight. Unlike in the first variation that counts how many times a character scores higher than each other, this method also considers how high a character scores against others.

For example, using the same score matrix in Table 2, the adjacency matrix for this second variation is as follows:

$$A = \begin{bmatrix} 0 & 2 & 4 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

The Borda Count for A, B, and C respectively are 8, 6, and 2. Thus, $A_{12} = 8 - 6 = 2$, $A_{13} = 8 - 4 = 4$, and so on. The process can be done with the following pseudocode (assuming that adjacency matrix A is initialized as a zero matrix).

```
i traversal[0..9]
    j traversal[0..9]
        diff ← arr_borda[i] - arr_borda[j]
        if diff > 0 then
            A[i][k] ← diff
```

In addition, for this variation, to ensure that there exists an eigenvalue for the adjacency matrix A, a small value ϵ is added to each element in the matrix. For this experiment, the value of ϵ is 10^{-5}.

IV. Implementation

The experiment in this paper is conducted with Python3 in a Jupyter Notebook environment (**.ipynb* file format). Python3 is chosen due to its high-level, readable code and abundant libraries with vast capabilities for data science such as NumPy and Pandas. Jupyter Notebook environment is chosen over a regular python file (**.py*) as it provides tools suitable for many data science purposes.

The general workflow and the corresponding code snippets for the experiment are written below. The full source code can also be accessed in Appendix A.

1. The popularity poll data from Love Live! fansite is compiled into an excel spreadsheet format (**.xlsx*)
2. The said spreadsheet is converted to a Pandas dataframe

```
[2]: df = pd.read_excel("nijigaku_popularity_poll.xlsx")
     df = df.drop(columns=["No."])
     df.head()
```

[2]:

	Ayumu Uehara	Kasumi Nakasu	Setsuna Yuki	Ai Miyashita	Emma Verde	Rina Tennoji	Kanata Konoe	Shizuku Osaka	Karin Asaka
0	4	3	2	9	7	1	8	6	5
1	6	7	1	9	5	2	4	3	8
2	6	4	1	7	8	2	9	3	5
3	9	4	2	7	8	3	6	5	1
4	8	4	1	5	9	7	3	6	2

*Figure 5. Converting Spreadsheet to Pandas Dataframe source: https://github.com/Nuetaari/Love-Live-Eigenvector-*Centrality

3. The said dataframe is converted from a dataframe of rankings to a dataframe of scores according to (2)

```
[5]: score_df = 9 - (df - 1)
     score_df.head()
```

[5]:

	Ayumu Uehara	Kasumi Nakasu	Setsuna Yuki	Ai Miyashita	Emma Verde	Rina Tennoji	Kanata Konoe	Shizuku Osaka	Karin Asaka
0	6	7	8	1	3	9	2	4	5
1	4	3	9	1	5	8	6	7	2
2	4	6	9	3	2	8	1	7	5
3	1	6	8	3	2	7	4	5	9
4	2	6	9	5	1	3	7	4	8

*Figure 6. Converting Matrix of Ranking to Matrix of Score source: https://github.com/Nuetaari/Love-Live-Eigenvector-*Centrality

4. The Borda Count is calculated into a NumPy array and the ranking results are shown

```
# borda count
score_arr = score_df.sum(axis=0).to_numpy()
print(score_arr)

[144 161 236 121  90 116 150 125 207]
```

*Figure 7. Calculating The Borda Count source: https://github.com/Nuetaari/Love-Live-Eigenvector-*Centrality

5. The score dataframe is converted into an adjacency matrix that represents how many times a character has a greater score compared to each other

```
[16]: # Variation 1

A1 = np.zeros((9,9))
score_matrix = score_df.to_numpy()

for i in range(30):
    for j in range(9):
        for k in range(9):
            if score_matrix[i][j] > score_matrix[i][k]:
                A1[j][k] += 1

print(A1)
```

```
[[ 0. 12.  4. 16. 23. 19. 12. 21.  7.]
 [18.  0.  3. 21. 24. 20. 17. 21.  7.]
 [26. 27.  0. 29. 28. 25. 25. 26. 20.]
 [14.  9.  1.  0. 21. 18. 11. 14.  3.]
 [ 7.  6.  2.  9.  0. 14.  9. 10.  3.]
 [11. 10.  5. 12. 16.  0. 12. 12.  8.]
 [18. 13.  5. 19. 21. 18.  0. 17.  9.]
 [ 9.  9.  4. 16. 20. 18. 13.  0.  6.]
 [23. 23. 10. 27. 27. 22. 21. 24.  0.]]
```

Figure 8. Variation 1 Adjacency Matrix Calculation source: https://github.com/Nuetaari/Love-Live-Eigenvector-Centrality

6. The eigenvector centrality on said adjacency matrix is calculated and the ranking results are shown

```
[17]: # Variation 1

eigenvalues1, eigenvectors1 = np.linalg.eig(A1)
print(eigenvalues1)
```

```
[104.24140207+0.j          -8.06011376+6.49130734j
  -8.06011376-6.49130734j -14.57493881+4.51750859j
 -14.57493881-4.51750859j -14.89532883+1.91450793j
 -14.89532883-1.91450793j -14.59031963+0.25919106j
 -14.59031963-0.25919106j]
```

```
[18]: norm = np.linalg.norm(eigenvectors1[:,0])
result1 = eigenvectors1[:,0] / norm
result1 = np.abs(result1.real)
print(result1)
```

```
[0.29056116 0.3275653  0.54762684 0.2267826  0.16476132 0.23975175
 0.31344951 0.24783443 0.46113988]
```

Figure 9. Variation 1 Eigenvector Centrality Calculation source: https://github.com/Nuetaari/Love-Live-Eigenvector-Centrality

7. The Borda Count NumPy array is converted into an adjacency matrix of difference between two characters' Borda Count results

```
[20]: # Variation 2

A2 = np.zeros((9,9))

for i in range(9):
    for j in range(9):
        difference = score_arr[i] - score_arr[j]
        if difference > 0:
            A2[i][j] = difference

print(A2)
```

```
[[  0.   0.   0.  23.  54.  28.   0.  19.   0.]
 [ 17.   0.   0.  40.  71.  45.  11.  36.   0.]
 [ 92.  75.   0. 115. 146. 120.  86. 111.  29.]
 [  0.   0.   0.   0.  31.   5.   0.   0.   0.]
 [  0.   0.   0.   0.   0.   0.   0.   0.   0.]
 [  0.   0.   0.   0.  26.   0.   0.   0.   0.]
 [  6.   0.   0.  29.  60.  34.   0.  25.   0.]
 [  0.   0.   0.   4.  35.   9.   0.   0.   0.]
 [ 63.  46.   0.  86. 117.  91.  57.  82.   0.]]
```

Figure 10. Variation 2 Adjacency Matrix Calculation source: https://github.com/Nuetaari/Love-Live-Eigenvector-Centrality

8. The eigenvector centrality on said adjacency matrix is calculated and the ranking results are shown

```
[21]: # Variation 2

epsilon = 0.00001
A2 += epsilon

eigenvalues2, eigenvectors2 = np.linalg.eig(A2)
print(eigenvalues2)
```

```
[ 4.5025866 +0.j          2.13526349+3.39751503j  2.13526349-3.39751503j
 -0.91207661+2.92168156j -0.91207661-2.92168156j -1.87305654+1.23246452j
 -1.87305654-1.23246452j -1.47134414+0.j         -1.73141315+0.j        ]
```

```
[23]: norm = np.linalg.norm(eigenvectors2[:,0])
result2 = eigenvectors2[:,0] / norm
result2 = np.abs(result2.real)
print(result2)
```

```
[7.15337495e-04 8.53430139e-03 9.92292247e-01 3.85714287e-05
 2.50337296e-06 1.69589965e-05 1.86600767e-03 9.01273928e-05
 1.23609302e-01]
```

Figure 11. Variation 2 Eigenvector Centrality Calculation source: https://github.com/Nuetaari/Love-Live-Eigenvector-Centrality

V. Results and Analysis

A. Results

Based on the experiment shown above, the result of the Borda Count $\boldsymbol{b}$, adjacency matrix A_1 and A_2 , normalized eigenvector $\boldsymbol{x_1}$ and $\boldsymbol{x_2}$ that correspond to the eigenvector centrality for each graph variation are as follows. Note that the character ordering for both rows and columns, in ascending index number, is Ayumu Uehara, Kasumi Nakasu, Setsuna Yuki, Ai Miyashita, Emma Verde, Rina Tennoji, Kanata Konoe, Shizuku Osaka, and Karin Asaka. For further analysis, the full ranking results are also available in Table 3.

$$\boldsymbol{b} = \begin{pmatrix} 144 \\ 161 \\ 236 \\ 121 \\ 90 \\ 116 \\ 150 \\ 125 \\ 207 \end{pmatrix}, \boldsymbol{x_1} = \begin{pmatrix} 144 \\ 161 \\ 236 \\ 121 \\ 90 \\ 116 \\ 150 \\ 125 \\ 207 \end{pmatrix}, \boldsymbol{x_2} = \begin{pmatrix} 144 \\ 161 \\ 236 \\ 121 \\ 90 \\ 116 \\ 150 \\ 125 \\ 207 \end{pmatrix}$$

$$A_1 = \begin{bmatrix} 0 & 12 & 4 & 16 & 23 & 19 & 12 & 21 & 7 \\ 18 & 0 & 3 & 21 & 24 & 20 & 17 & 21 & 7 \\ 26 & 27 & 0 & 29 & 28 & 25 & 25 & 26 & 20 \\ 14 & 9 & 1 & 0 & 21 & 18 & 11 & 14 & 3 \\ 7 & 6 & 2 & 9 & 0 & 14 & 9 & 10 & 3 \\ 11 & 10 & 5 & 12 & 16 & 0 & 12 & 12 & 8 \\ 18 & 13 & 5 & 19 & 21 & 18 & 0 & 17 & 9 \\ 9 & 9 & 4 & 16 & 20 & 18 & 13 & 0 & 6 \\ 23 & 23 & 10 & 27 & 27 & 22 & 21 & 24 & 0 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & 0 & 0 & 23 & 54 & 28 & 0 & 19 & 0 \\ 17 & 0 & 0 & 40 & 71 & 45 & 11 & 36 & 0 \\ 92 & 75 & 0 & 115 & 146 & 120 & 86 & 111 & 29 \\ 0 & 0 & 0 & 0 & 31 & 5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 26 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 29 & 60 & 34 & 0 & 25 & 0 \\ 0 & 0 & 0 & 4 & 35 & 9 & 0 & 0 & 0 \\ 63 & 46 & 0 & 86 & 117 & 91 & 57 & 82 & 0 \end{bmatrix}$$

Table 3. Experiment Results

Character Name	Borda Count		Variation 1		Variation 2	
	Normalized Borda Count	Ranking	Eigenvector Centrality	Ranking	Eigenvector Centrality	Ranking
Ayumu Uehara	0.307	5	0.291	5	7.15×10^{-4}	5
Kasumi Nakasu	0.344	3	0.328	3	8.53×10^{-3}	3
Setsuna Yuki	0.504	1	0.548	1	0.992	1
Ai Miyashita	0.258	7	0.227	8	3.9×10^{-5}	7
Emma Verde	0.192	9	0.165	9	3×10^{-6}	9
Rina Tennoji	0.248	8	0.240	7	1.7×10^{-5}	8
Kanata Konoe	0.320	4	0.313	4	1.87×10^{-3}	4
Shizuku Osaka	0.267	6	0.248	6	9×10^{-5}	6
Karin Asaka	0.442	2	0.461	2	0.124	2

B. Analysis

From Table 3, we can see that the ranking results are mostly consistent across all methods. The ranking order of fan-favorite Love Live! Nijigasaki High School Idol Club characters in descending order are Setsuna Yuki, Karin Asaka, Kasumi Nakasu, Kanata Konoe, Ayumu Uehara, Shizuku Osaka, Ai Miyashita/Rina Tennoji, and Emma Verde. Though the results are similar, more analysis can be done through the normalized Borda Count and the eigenvector centralities which act as the measurement methods for the characters' ranking.

Table 4. Experiment Results' Statistics

	Normalized Borda Count	Eigenvector Centrality	
		Variation 1	Variation 2
Max Value	0.504	0.548	0.992
Min Value	0.192	0.165	3×10^{-6}
Range	0.312	0.383	0.992
Skewness	0.712	0.865	2.407

Results from Table 3 and Table 4 shown above provide more insights on how each measurement metric evaluates the character rankings. The normalized Borda Count result shows a relatively balanced distribution, with values ranging from 0.192 (Emma Verde) to 0.504 (Setsuna Yuki), meaning that it has a moderate value range of 0.312. This suggests that this measurement metric captures a diverse range of preferences without extreme outliers. This balanced nature of the result implies that the Borda Count ranking method is well-suited for measuring overall popularity in a more general context since it reflects preferences consistently across all characters with no extreme outliers.

In contrast, variation 1 of the eigenvector centrality method shows a similar pattern to the Borda Count, with values ranging from 0.165 (Emma Verde) to 0.548 (Setsuna Yuki), meaning that this method resulted in a value range of 0.383. This broad value range compared to the Borda Count method suggests that the variation 1 of the eigenvector centrality method, which represents the score matrix as an adjacency matrix of how many times a character scores higher than other characters, is also effective at capturing a general popularity ranking measurement. Furthermore, the difference in results in the lower ranking (Ai Miyashita and Rina Tennoji) may arise from the adjacency matrix representation of this method. The adjacency matrix captures a pairwise comparison between characters rather than a broad, direct comparison done by the Borda Count method that might produces different result and insights.

On the other hand, variation 2 of the eigenvector centrality method shows a highly skewed distribution, though the ranking result is identical to the Borda Count method. The value ranges from 3×10^{-6} (Emma Verde) to 0.992 (Setsuna Yuki), an extreme value range of ≈0.992. Notice that besides the first place Setsuna Yuki's eigenvector centrality, other characters' eigenvector centrality has a near-zero value. This is also represented by the high skewness of this data, 2.407, which is approximately three times higher than both the Borda Count method and the first variation of the eigenvector centrality method. This means that this measurement method is more useful for identifying the most dominant character (or, in this context, the most popular character) rather than a balanced view of overall popularity.

In summary, Borda Count and variation 1 of the eigenvector centrality emphasize more on a balanced and more general influence, providing a result that represents a general context of popularity measurement. In contrast, variation 2 of the eigenvector centrality focuses more on the dominance of a single character. This means that this method is more useful and suitable for identifying a singular, most popular character rather than a ranking order for the characters.

VI. Conclusion

This paper aimed to evaluate fan-favorite Love Live! Nijigasaki High School Idol Club characters based on popularity poll ranking through eigenvector centrality method. In this paper, two graphs in adjacency matrix representation were used: a directed weighted graph of how many times a character scores higher compared to each other (variation 1) and a directed weighted graph of the difference between two characters' Borda Count results. Both variations resulted in an almost consistent ranking compared to Borda count ranking result as its base comparison, with the top three characters being Setsuna Yuki, Karin Asaka, and Kasumi Nakasu.

Furthermore, the experiment results showed that differences in graph representation may result in differences in the eigenvector centrality values of each vertex/entity. Variation 1 resulted in a balanced distribution while variation 2 resulted in a highly skewed distribution. This finding suggests that eigenvector centrality can be used for different purposes: general popularity evaluation (variation 1) and identifying a single standout entity in a social network (variation 2).

This study focuses more on exploring eigenvector centrality as a ranking method rather than exploring which ranking methods are best used for some given scenario. Future research could explore further applications of eigenvector centrality with the graph representation used in this paper or develop a more effective graph representation, testing eigenvector centrality on a larger and more varied dataset to analyze the method's general properties and behaviors, and comparing said methods with other common ranking methods such as Condorcet method and PageRank.

VI. Appendix

A. Appendix A

The excel spreadsheet of the popularity poll results and the full source code of the experiment can be found on this github repository:

*https://github.com/Nuetaari/Love-Live-Eigenvector-C*entrality

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Hereby I declare that this paper that I have written is my own work, not a reproduction or translation of someone else's work, and not plagiarized.

Bandung, 31 December 2024

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